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A STUDY ON FRACTAL MATHEMATICAL MODELS

MD. MEGANUR RHAMAN[1] & A. H. M. ZADIDUL KARIM[2]

[1]Ahsanullah University of Science and Technology (AUST), Dhaka, Bangladesh

[2]University of Asia Pacific (UAP), Dhaka, Bangladesh

ABSTRACT

Premature ventricular contractions (PVC) are premature heartbeats originating from the ventricles of the heart. These heartbeats occur before the regular heartbeat. The Fractal analysis is most mathematical models produce intractable solutions. Some studies tried to apply the fractal dimension (FD) to calculate of cardiac abnormality. Based on FD change, we can identify different abnormalities present in ECG. Present of the uses of Poincare plot indexes and the sample entropy (SE) analyses of heart rate variability (HRV) from short term ECG recordings as a screening tool for PVC. Poincare plot indexes and the SE measure are used for analyzing variability and complexity of HRV. A clear reduction of standard deviation (SD) projections in Poincare plot patterns is observed a significant difference of SD between healthy Person and PVC subjects. Finally a comparison is shown for FD, SE and Poincare plot parameters.

KEYWORDS: ECG, FD, HP, HRV, IHR**,** MIT-BIH, PSD, PVC, RD, RS, SD, SE

1. INTRODUCTION

Premature ventricular contractions (PVC) are early or extra heartbeats that commonly occur and usually harmless in normal hearts, but can cause problems in hearts with pre-existing diseases. PVC may be perceived as a "skipped beat" or felt as palpitations in the chest. In a normal heartbeat, the ventricles contract after the atria have helped to fill them by contracting, in this way the ventricles can pump a maximized amount of blood both to the body and to the lungs [1]. The ventricle electrically discharges prematurely before the normal electrical discharges. These premature discharges are due to electrical "irritability" of the heart muscle of the ventricles and can be caused by heart attacks, electrolyte imbalances, lack of oxygen, or medications. Immediately after a PVC, the electrical system of the heart resets, this resetting causes a brief pause in heartbeat and some patients report feeling the heart briefly stopping after PVC. PVC may be more common among older persons, patients with high blood pressure, and patients with heart diseases. PVC can also occur in young healthy individuals without known heart diseases or high blood pressure. Patients with three or more consecutive PVC in a row may develop ventricular tachycardia [1]. Conventionally used time and frequency domain parameters of HRV [5, 6] are not always suitable for analysis because of the non-stationary characteristic of the ECG. The visual analysis of variability of the Poincaré plot [7] and quantification of the unpredictability and complexity of the heart rate using sample entropy [8] are being increasingly used because they can be computed from shorter ECG records.

Fractal is a mathematical investigation for characterizing complex, replicating geometrical patterns at different scale lengths [2]. Fractal behaviour is exhibited by the heart in electrocardiogram signals and by the brain in electroencephalogram (EEG) signals [3], [4]. This paper presents the application of fractal theory, Poincare plot and SE method to the analysis of ECG data. The PVC cardiac abnormalities are discussed in this paper. The ECG's are taken for 30 minutes and sampled at 360Hz. At first FD of healthy persons (HP) are determined by the three methods. These three methods are then applied for patients with PVC diseases to determine its range of variation from healthy patients. So by

determining the FD of an ECG signal, an estimation of heart condition can be made.

The aim of this study was to determine how and which of the variability and complexity parameters of the HRV derived from the FD, Poincare plots and sample entropy are different in patients with PVC compared with subjects with normal rhythm.

2. METHODS

The ECG signal will be processed through a series of steps to calculate fractal dimension (FD). Different methods of calculating FD such as relative dispersion (RD) analysis, power spectral density (PSD) analysis, and rescaled range (RS) analysis [12], [13] will be applied. An algorithm based on indexes of Poincaré plots and sample entropy, developed to distinguish between ECG of HP and PVC subjects. The data sets of ECG are taken from MIT-BIH arrhythmia database.

2.1 Relative Dispersion (RD) Analysis

The basic principle of RD analysis making estimates of the variance of the signal at each of several different levels of resolution form the basis of the technique; for fractal signals a plot of the log of the standard deviation versus the log of the measuring element size gives a straight line with a slope of 1 - D, where D is the fractal dimension. Studies were done known signals with a specified value for H, the characterizing Hurst coefficient. H is a measure of roughness; the roughness in the signal is maximal at H near zero. White noise with zero correlation has H = 0.5. Smoother correlated signals have H near 1.0. For one-dimensional series, H = 2 - D, where D is the fractal dimension, 1<D<2.

$$\text{Standard deviation (SD)} = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{N}$$

Where, x_i = Random variable, $\bar{x}$ = Mean of the variables, N= Number of Samples.

By calculating the RD for different bin sizes, n and fitting the square law function:

$$RD = RD_0\left(\frac{n^{H-1}}{n_0}\right)$$

Where, RD_0 is the RD for some reference bin size n_0.

The whole data set is used for each calculation of RD (n) at each level of resolution or number of pieces, n. The exponent can be best estimated by a log-log transformation.

$$\log(RD) = \log(RD_0) + (H-1)\log\left(\frac{n}{n_0}\right)$$

Here, H-1 = slope, for fractal dimension, D=2-H.

$$D = 2 - (1 + Slope)$$

So, $D = 1 - Slope$

2.2 Power Spectral Density

The power spectrum of a pure fractional Brownian motion is known to be described by a power law function:

$$|A|^2 = \frac{1}{f^{\beta}}$$

Where |A| is the magnitude of the spectral density at frequency, with an exponent β = 2H + 1. Here again, a straight line is fitted from a log-log plot and H is calculated from the slope β. In the frequency domain, fractal time series exhibit power law properties, $P(f) = f^{-\alpha}$

Where P(f) is the power spectral density f and the exponent α is the so called power-spectral index. For the values region between FD = 1 and FD = 2 the following relationship between FD and α is valid,

$FD = \frac{(5-\alpha)}{2}$, for $1 < FD < 2$

Fractal dimension, $D = \frac{(5 - Slope)}{2}$

2.3 Rescaled Range Analysis

The basis of the RS analysis was laid by Hurst [2]. Mandelbrot and Wallis examined and further elaborated the method. Feder [10] gives an overview of theory and applications, and adds some more statistical experiments. There are two factors used in this analysis: firstly the range R, this is the difference between the minimum and maximum 'accumulated' values or cumulative sum of X(t, τ) of the natural phenomenon at discrete integer-valued time t over a time span τ and secondly the standard deviation S, estimated from the observed values $X_i(t)$. Hurst found that the ratio R/S is very well described for a large number of natural phenomena by the following experiential relation

$$\frac{R(\tau)}{S(\tau)} \propto \tau^{H}$$

Where the coefficient c was taken equal to 0.5 by Hurst. R and S are defined as

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau)$$

And, $S(\tau) = \left(\frac{1}{\tau} \sum_{t=1}^{\tau} \left[\xi(t) - \langle \xi \rangle_{\tau} \right]^2 \right)^{\frac{1}{2}}$

Where: $\langle \xi \rangle_{\tau} = \frac{1}{\tau} \sum_{1}^{\tau} \xi(t)$

And, $X(t,\tau) = \sum_{N=1}^{t} \left[\xi(u) - \langle \xi \rangle_{\tau} \right]$

We calculate the individual calculations for each interval length. A straight line is fitted in the log-log plot:

$$\log\left[\frac{R(\tau)}{S(\tau)}\right] = c + H \log(\tau)$$

Where H = slope. So, Fractal dimension, $D = 2 - H$ or $D = 2 - Slope$.

2.4 Poincare Plot Analysis

The Poincare plot is a two-dimensional visualization display of the dynamic properties of a system from a time series [9]. The Poincare plot was generated as a scatter plot of current instantaneous heart rate (IHR) against the IHR immediately preceding it [11].

In this paper we define the Poincare plot for a data vector $RR_i = (RR_1, RR_2 \ldots RR_N)$ of length N. First, we define two auxiliary vectors:

$$RR_i^+ = (RR_1, RR_2, \ldots\ldots\ldots, RR_{N-1})$$

$$RR_i^- = (RR_2, RR_3, \ldots\ldots\ldots, RR_N)$$

The Poincare plot consists of all the ordered pairs:

$$(RR_i^+, RR_i^-), \qquad i = 1 \ldots\ldots\ldots N-1.$$

SD1 and SD2 are two standard Poincare plot descriptors. SD1 is defined as the standard deviation of projection of the Poincaré plot on the line perpendicular to the line of identity (y = −x) while SD2 as that on the line of identity (y = x). We can define SD1 and SD2 as:

$$SD1 = \sqrt{Var(X_1)}$$

$$SD2 = \sqrt{Var(X_2)}$$

Where,

$$X_1 = \frac{(RR_i^+ - RR_i^-)}{\sqrt{2}}$$

$$X_2 = \frac{(RR_i^+ + RR_i^-)}{\sqrt{2}}$$

We define a parameter S which reflects the total variability of the Poincaré plot which is the area of the ellipse S

$$S = \pi \times SD1 \times SD2$$

2.5 Sample Entropy Analysis

Entropy is related to dynamical systems and is defined as the rate of information production. SE entropy measure frequently applied to clinical cardiovascular and other time series analysis of different types of abnormalities [8, 14]. Quantification of the irregularity and difficulty of the heart rate using sample entropy [14, 11] are increasingly used because they can be computed from shorter HRV records. Using SE statistics a sequence of total N numbers of IHR such as IHR (1),

IHR (2),……, IHR (N). To compute SE of each IHR data set, m-dimensional vector sequences p_m (i) were constructed from the IHR time series $p_m(1), p_m(2),..... p_m(N-m+1)$, where the index i can take values ranging from 1 to N-m+1. If the distance between two vectors $p_m(i)$ and $p_m(j)$ is defined as $|p_m(i) - p_m(j)|$, then,

$$C_i^m(r) = \frac{M}{N-m+1}$$

where, M is the number of vectors such that $|p_m(i) - p_m(j)| < r$ for $i \neq j$, m specifies the pattern length which is 2 in this study, r defines the criterion of similarity which was varied from 10~90% of the standard deviation of IHR data (N=2000 beats). $C_i^m(r)$ is considered as the mean of the fraction of patterns of length m that resemble the pattern of the same length that begins at index i. The SE is calculated by the following equation,

$$SE(N,m,r) = \ln\left(\frac{\Sigma_{i=1}^{N-m-1} C_i^m(r)}{\Sigma_{i=1}^{N-m} C_i^m(r)}\right)$$

3. RESULTS AND DISCUSSIONS

Figures 1, 2 and 3 show the RD, PSD and RS analysis for data 115 with data length N=4096. Table 1 summarizes the result from RD, PSD and RS for HP and PVC. Here the values of seven data sets (MIT-BIH data set # 115, 117, 122, 100, 105, 111 and 116) of HP ECG are computed. Same is done for seven data sets (119, 208, 221, 106, 201, 210 and 233) of PVC patients. The FD descriptors are analyzed to see if any significant difference is found between HP and PVC data series. Figures 4 and 5 show the Poincaré plot and IHR time series of normal and PVC data sets. Here, the average values of 8 data sets (100, 105, 111, 112, 116, 118, 121 and 122) of HP ECG are computed. Same is done for rest 8 data sets (106, 119, 201, 208, 210, 221, 223 and 233) of PVC patients. The SD descriptors are analyzed to see if any significant difference is found between normal and PVC data series. Figure 5 demonstrates the change of SE with m=2 and r=0.1*SD to 0.9*SD of IHR data for normal and PVC subjects. The mean values of SE of the healthy group were found to be lower than that of PVC at all r values except at 0.1*SD, 0.2*SD and 0.3*SD. Statistically, the SE of HP group is found to be significantly different from that of PVC group at r > 0.3*SD. Figure 6 display the average sample entropy of normal and PVC beats.

From Table 1, it is obvious that there is a clear reduction of FD for RD, PSD and RS in PVC data series. From Table 2 we found different range of FD with RD, PSD and RS for HP and PVC. For RD the range of PVC (1.38-1.51) falls in HP (1.34-1.54). For PSD the range of PVC (1.69-1.74) is lower than HP (1.69-1.90). For RS the range of HP (1.63-1.71) is higher than the ranges of PVC (1.48-1.53). Table 3 signify the FD range variation for different beat type and its difference with different methods. From Table 4 it is obvious that there is a clear reduction of SD1, SD2, Ratio and S in the HP and PVC. The average SE values at different ‘r’ are summarized in Table 5.

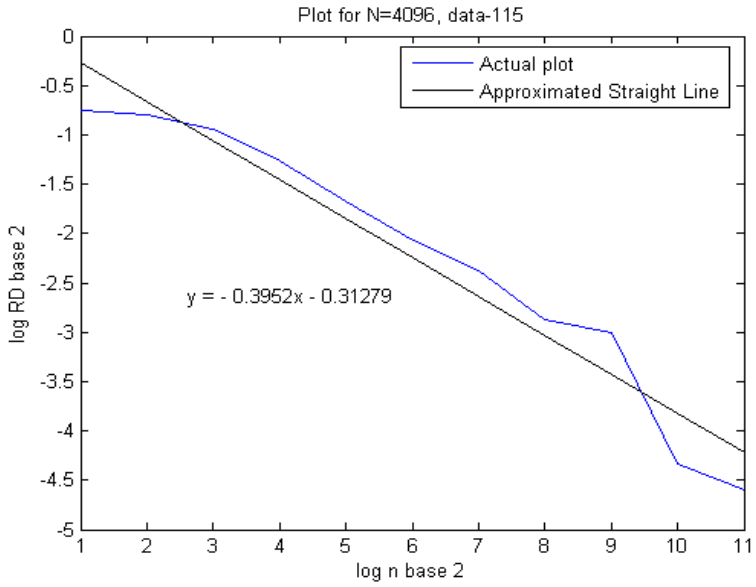

Figure 1: Actual and Approximated Straight Line for RD Analysis for Data Set-115

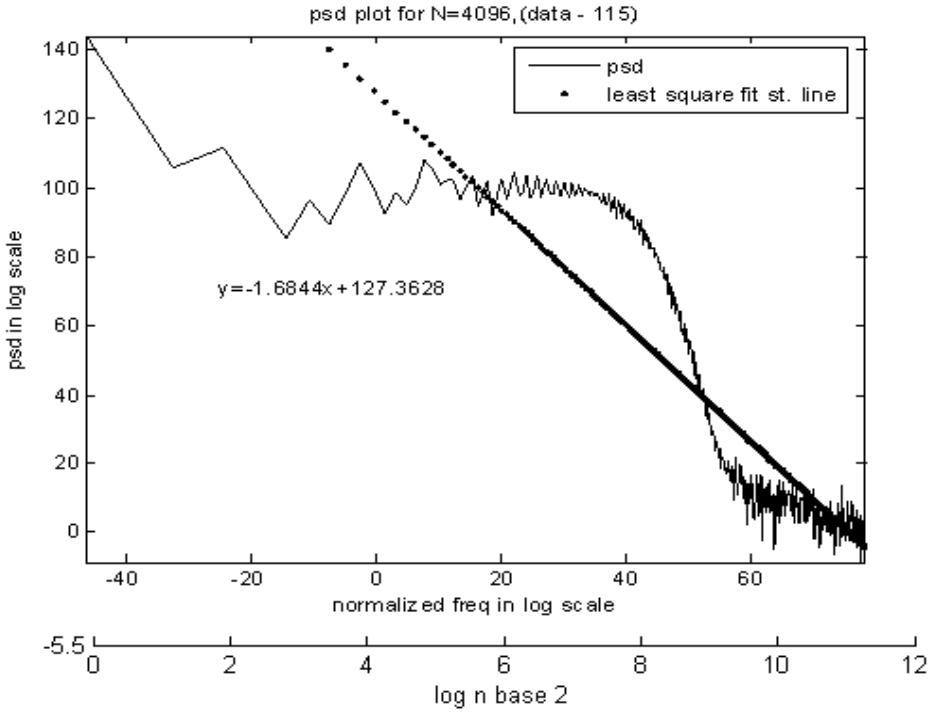

Figure 2: Actual and Approximated Straight Line for PSD Analysis for Data Set-115

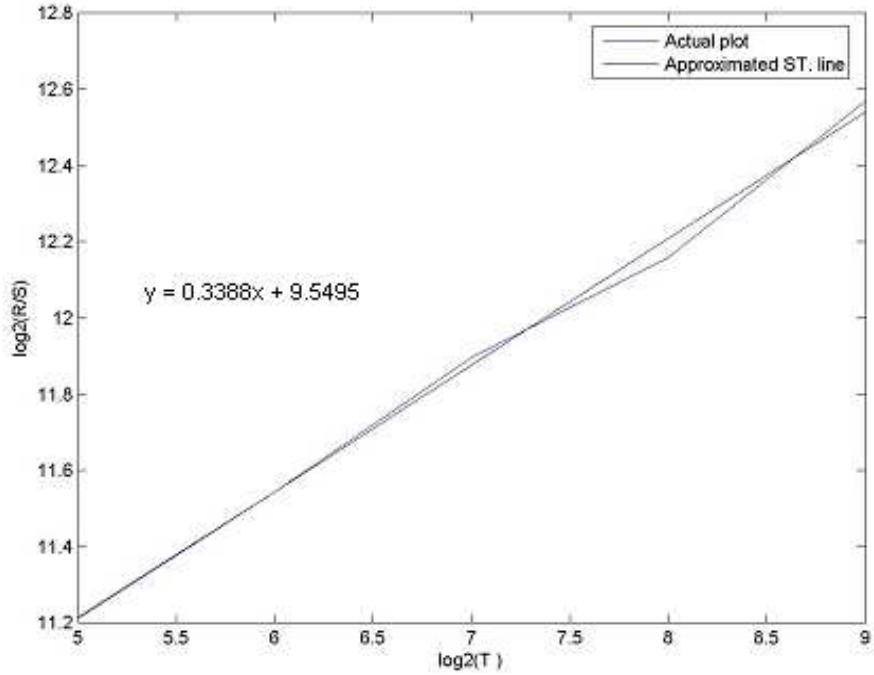

Figure 3: Actual and Approximated Straight Line for RS Analysis for Data Set-115

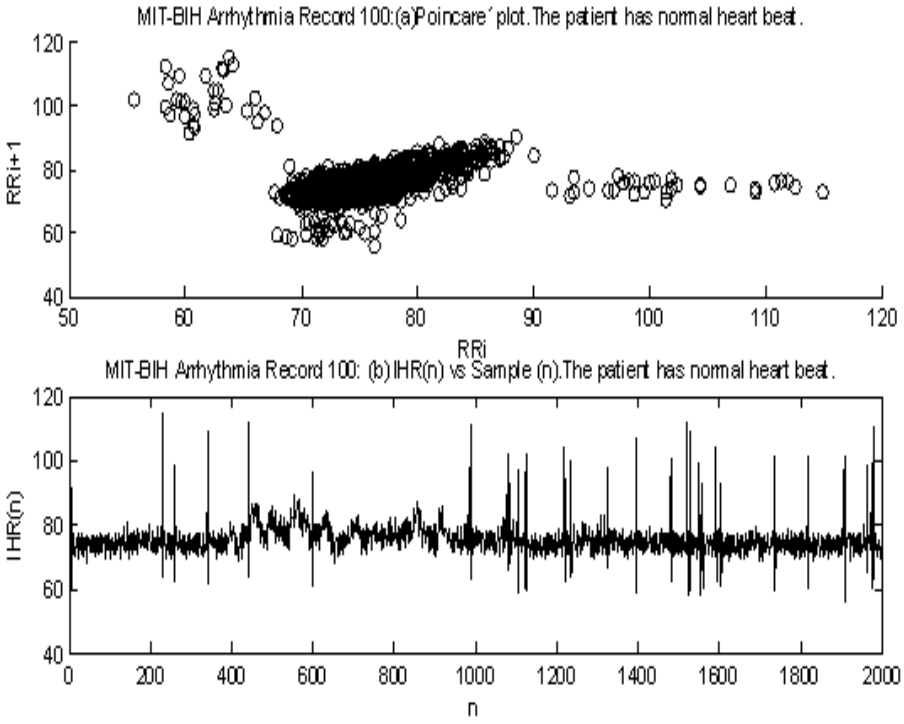

Figure 4: (a) Poincare Plot and (b) IHR of MIT-BIH Record_100 for HP

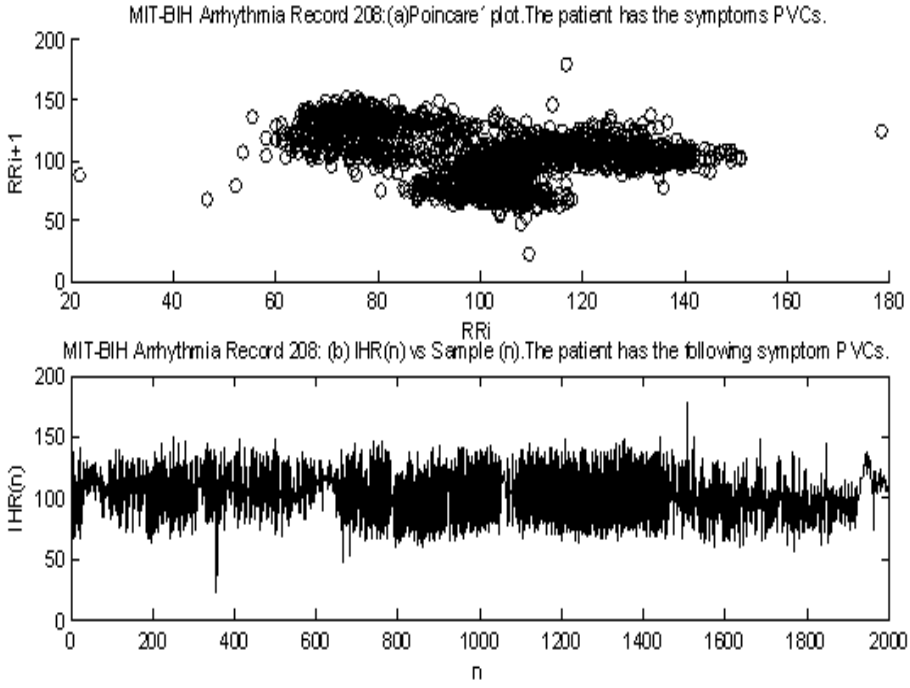

Figure 5: (a) Poincare Plot and (b) IHR of MIT-BIH Record_208 for PVC

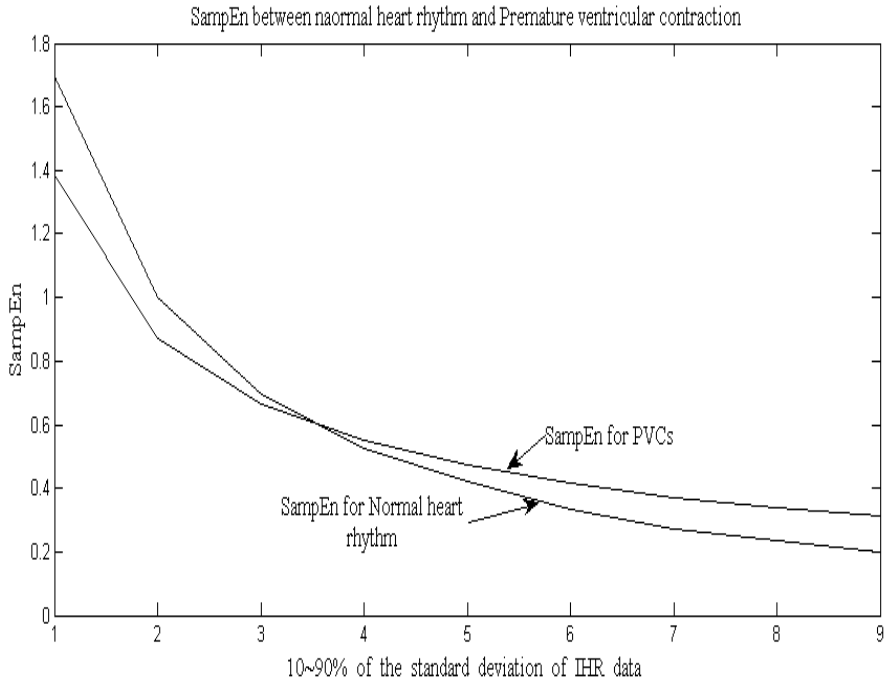

Figure 6: Average Sample Entropy of Normal and PVC Beats

Table 1: FD of Different Beat Type for Different Methods When Data Length N=4096

Beat Type	Record No	FD for RD	FD for PSD	FD for RS
HP	115	1.3952	1.7965	1.6612
	117	1.5349	1.8148	1.6626
	122	1.4460	1.7704	1.6521
	100	1.3492	1.8975	1.7012
	105	1.4309	1.7142	1.6925
	111	1.3861	1.6905	1.7001
	116	1.4258	1.7709	1.6392
PVC	119	1.4302	1.7015	1.4836
	208	1.4312	1.7159	1.5146
	221	1.3975	1.7147	1.5256
	106	1.4202	1.7215	1.5236
	201	1.5012	1.7009	1.5046
	210	1.4975	1.6947	1.5256
	233	1.3802	1.7305	1.5136

Table 2: Range of FD for Different Beat Type for Different Method When N=4096

Beat Type	FD Range for RD	FD Range for PSD	FD Range for RS
HP	1.34-1.54	1.69-1.90	1.63-1.71
PVC	1.38-1.51	1.69-1.74	1.48-1.53

Table 3: FD Range Variation for Different Beat Type and Its Difference

Beat Type	FD range Variation for RD	FD range Variation for PSD	FD range Variation for RS
HP	0.186	0.207	0.040
PVC	0.121	0.234	0.042
Difference	0.065	0.027	0.002

Table 4: Average Values of Poincaré Plot Parameters

Parameter	Normal Rhythm	PVC
SD1	5.61	24.91
SD2	7.14	20.44
Ratio	0.74	1.31
Area of ellipse(S)	150.48	1607

Table 5: Average SE Values of HP And PVC Subjects With (M = 2)

Similarity Criterion ®	SE for HP	SE for PVC Data
10% of SD	1.69	1.38
20% of SD	1.00	0.87
30% of SD	0.69	0.66
40% of SD	0.52	0.55
50% of SD	0.42	0.47
60% of SD	0.34	0.41
70% of SD	0.27	0.37
80% of SD	0.23	0.34
90% of SD	0.20	0.31

4. CONCLUSIONS

This work describes the application of fractal theory to heart rate dynamics. The fractal dimension for HP hearts as well as hearts with PVC is calculated here. We compared three numerical methods to estimate the fractal dimension. RD analysis provides good result for longer data length, for PSD analysis the range of FD is close to the range of healthy person. We found better result for RD analysis because the difference between normal value and PVC is higher for RD data. The sample entropy (SE) and Poincare plot analysis to differentiate the normal rhythm from the PVC. Poincare plot can provide supplementary information about beat to beat HRV structure which cannot be obtained by conventional time and frequency domain analysis [15]. Poincaré plot images represent short and long-term variability. The results show that there is a significant difference between the Poincaré plot parameters of normal rhythm data sets and that of PVC data sets. Lower values of SE reflect more regular time series while higher values are associated with less predictable (more complex) time series [16]. The major finding of this study shows a lower SE is found for normal rhythm data sets and higher in PVC data sets. To make these methods applicable, works are needed to be done with all normal heart abnormalities, with a large number to samples for each abnormality. The nonlinear measures can be investigated further in future studies of heart rate variability in different cardiac diseases. The same methods can be used for analysis of two or three dimensional signals, the methods can be extended to account for anisotropy, making the analysis more complicated but adhering to the same basic theory. The fractal analysis can also be applied further in image analysis, fluid dynamics, investing natural phenomenon etc.

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